Physicochemical Problems of Mineral Processing, 41 (2007), 397-413
Fizykochemiczne Problemy Mineralurgii, 41 (2007), 397-413

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EFFECT OF PARTICLE SHAPE ON JIG SEPARATION EFFICIENCY

Received May 15, 2007; reviewed; accepted July 4, 2007

Settling velocity of particles, which is the main parameter of jig separation, is affected by physical (density) and the geometrical (size and shape) properties of particles. The authors worked out a calculation algorithm of particles falling velocity distribution for monosized spherical and irregular particles assuming that the density of particles, their size and shape constitute random variables of fixed distributions. The distributions of falling velocity of irregular particles of a narrow size fraction were calculated utilizing industrial experiments. The measurements were executed and the histograms of distributions of projection diameter, as well as volume and dynamic shape coefficient, were drawn. The separation accuracy was measured by the change of process imperfection of irregular particles in relation to spherical ones, resulting from the distribution of particles settling velocity.

Key words: spherical particles, irregular particles, settling velocity, imperfection, enrichment, jig, random variable distribution

INTRODUCTION

Enrichment in jigs occurs during vertical pulsating motion of the suspension containing particles. After some time of such motion, the particles are stratified into groups differing in physical (density) and geometrical (particle size and shape coefficient) properties. According to Mayer's potential theory (Mayer, 1964) the separation runs towards minimizing the potential energy of the system. If the material were separated into densimetric fractions and each densimetric fraction into size fraction and the size fractions arranged from the largest to the smallest, then the porosity of the layer after separation would be larger than before separation (Kuprin et al, 1983). Consequently, the potential energy of the system after separation would be larger than before separation. This is in contradiction with the principle of least action according to which the processes run towards the minimization of potential energy.

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The decrease of porosity will occur when the voids between larger particles are replaced by smaller ones. It will happen in the situation when smaller particles of higher density will be transferred to the higher sub-layer of larger particles of lower density. Such a stratification means that in consecutive layers, for ideal separation, there are particles of the same value of terminal settling velocity. Thus, it can be stated that terminal settling velocity is this property (separation argument) according to which, for ideal separation, the stratification of the material into respective subsets of the same value of separation argument occurs.

In industrial separation processes, as a result of particles dispersion resulting from interactions between particles, the fouling of respective subsets with particles of neighboring layers occurs, and these are characterized by different, for a given layer, values of settling velocity. This phenomenon is characterized by the partition curve and its mathematical representation, i.e. the partition function while numerically it is estimated by the so-called probable error and imperfection parameters. These indicators are a measure of separation efficiency which determines the quality of separation machinery. Respectively, for every separator the representative measure of separation efficiency should be applied, corresponding to the real separation argument in the separator. Since the settling velocity of an irregular particle depends on the particle shape, this paper presents the method of evaluation of the effect of particle shape on separation efficiency. To achieve this, the authors calculated the distributions of settling velocities of spherical and irregular particles according to the empirical data for a narrow 8-10 [mm] size fraction.

TERMINAL SETTLING VELOCITY OF IRREGULAR PARTICLES

Terminal settling velocity of a particle is the value of velocity which is obtained by a particle which moves uniformly in the medium. It is calculated from the particle motion equation. The following forces act on irregular particles moving in water under the force of gravity for Reynolds numbers greater than $5\cdot10^2$:

1) gravity force: $Q = \rho V g$ (1)

2) buoyancy force: $F_w = \rho_o V g$ (2)

3) medium dynamic resistance force, determined by Newton's formula:

$$P = -\psi_z \frac{1}{2}\rho_0 v_t^2 S, \tag{3}$$

where: V - particle volume, v_t – momentary velocity of particle motion, ψ_z – drag coefficient for a particle, S – particle projection area on the plane perpendicular to the motion direction, ρ – particle density, ρ_o – medium density, g – acceleration due to gravity, $\mathrm{Re} = \frac{\rho_o v_t r}{\eta}$ – Reynolds number, r – particle radius, η – dynamic viscosity coefficient of the medium. The expression $\frac{1}{2}\rho_0 v^2$ represents the liquid hydrodynamic pressure. Respectively, the equation of particle motion in a vertical direction will be as follows:

$$\rho V \frac{dv}{dt} = (\rho - \rho_0)Vg - \psi_z \frac{1}{2}\rho_0 v_t^2 S \tag{4}$$

Equation (4), after transformation, takes the form:

$$\frac{dv}{dt} = a - bv_t^2 \tag{5}$$

where $a = \frac{\rho - \rho_0}{\rho} g$, $b = \frac{\psi_z \rho_0 S}{2\rho V}$.

Formula (5) is known as Riccati's equation (Leja, 1971). This equation can be solved by quadratures and its solution is as follows (Ponomariev, 1973):

$$v_t = \sqrt{\frac{a}{b}}\, tgh\left(\sqrt{ab}\, t\right) = \sqrt{\frac{2(\rho - \rho_0)Vg}{\psi_z \rho_0 S}}\, tgh\left(\sqrt{\frac{\psi_z \rho_0 (\rho - \rho_0) g S}{2\rho^2 V}}\, t\right) \tag{6}$$

The following limit:

$$\lim_{t\to\infty} v_t = \sqrt{\frac{2(\rho - \rho_0)Vg}{\psi_z \rho_0 S}} = v \tag{7}$$

presents the general formula for the terminal settling velocity of a particle. Its detailed form, with respect to irregular particles, must consider particle shape, characterized by shape coefficient and the value of drag coefficient. In Eq. 7, the particle drag coefficient ψ_z, particle volume V and particle projection area S occur. The following measures of particle size are used to characterize these values:

- equivalent diameter d_z, which is the diameter of a sphere of the volume equal to the particle volume V according to the dependence:

$$d_z = \sqrt[3]{\frac{6V}{\pi}} \tag{8}$$

- projection diameter d_p, which is equal to the diameter of a circle of the area equal to the particle projection area S, i.e.

$$d_p = \sqrt{\frac{4S}{\pi}} \tag{9}$$

The particle drag coefficient is connected with the sphere drag coefficient in the Newtonian range of Reynolds number by the following dependence (Thomson and Clark, 1991, Ganser, 1993):

$$\psi_z = k_2 \psi_k \tag{10}$$

where: k_2 is the dynamic or Newtonian coefficient of particle shape.

For the sphere, the drag coefficient is ψ_k = 0.46 (Abraham, 1970, Concha and Almendra, 1979).
Thus:

$$\psi_z = 0.46\, k_2\,. \tag{11}$$

Therefore, the particle drag coefficient depends on the particle shape and more irregular particles provide higher resistance forces. Ganser (1993) gave the following statistical dependence of dynamic shape coefficient upon the particle sphericity coefficient ϕ:

$$k_2 = 10^{1.148\,(-\log\phi)^{0.5743}}\,. \tag{12}$$

Equation 12, according to Ganser, can be applied to particles of any shape while the sphericity coefficient as the particle measure is defined as follows (Wadell, 1933, according to Heiss and Coull, 1952):

$$\phi = \left(\frac{d_z^2}{d_k^2}\right) = \left(\frac{A_S}{A}\right)_v \tag{13}$$

where: d_k – diameter of a sphere of the area equal to the particle area, A – particle area, A_s – area of a sphere of the volume equal to the particle volume.

Due to the difficulties in measuring the particle area, Wadell proposed an approximation of the sphericity coefficient by the circularity coefficient, calculated from measurements on the plane and defined as follows:

$$\phi \cong k_c = \left(\frac{C}{C_z}\right)_s \tag{14}$$

where: C_z – perimeter of the particle projection area, C – perimeter of the circle of the area equal to the area of the particle projection area.

If appropriate expressions of formulas in Eqs 11, 8, and 9 are substituted for ψ_z , V and S into formula in Eq.7, the following expression for irregular particle settling velocity is obtained:

$$v = 5.33\sqrt{x}\,\frac{\sqrt{d_z^3}}{d_p\sqrt{k_2}} \tag{15}$$

where : $x = \dfrac{\rho - \rho_0}{\rho_0}$ – reduced particle density.

For the spherical particle $d_z = d_p = d$ and $k_2 = 1$, thus the settling velocity is expressed by the formula:

$$v = 5.33\sqrt{x}\,\sqrt{d}\ . \tag{16}$$

Heywood (1937, according to Heiss and Coull, 1952) proposed to determine the particle volume by the volume shape coefficient k_{1H}, according to the formula:

$$V = k_{1H} d_p^3 . \tag{17}$$

In this paper the volume shape coefficient k_1 was determined from the analogical expression:

$$V = k_1 \frac{\pi d_p^3}{6}\ . \tag{18}$$

After substituting ψ_z, S and V with the expression from Eq. 11, 9, and 18 for irregular particle settling velocity into Eq.7, the following formula is obtained:

$$v = 5.33\sqrt{x}\,\sqrt{d_p}\,\sqrt{\left(\frac{k_1}{k_2}\right)} \tag{19}$$

Equations 16 and 19 will be applied further in this paper to determine the distributions of settling velocities of spherical and irregular particles.

DISTRIBUTION OF SETTLING VELOCITY FOR A SAMPLE OF MONOSIZED SPHERICAL PARTICLES

Let R, X, D_p, K_1, and K_2 determine random variables of particle density, reduced density, particle projection diameter, particle volume shape coefficient and particle dynamic shape coefficient, respectively. For spherical particles $k_1 = k_2 = 1$, $d_p = d_o$ where d_o – size of feed particles. In this situation particle settling velocity v_{ms} is expressed by the formula:

$$v_{ms} = 5.33\sqrt{x}\sqrt{d_o} \tag{20}$$

Reduced density x is connected with particle density by the dependence:

$$x = \frac{\rho - \rho_o}{\rho_o} \tag{21}$$

from which:

$$\rho = \rho_o x + \rho_o = \rho(x) \tag{22}$$

Let the random variable R have the distribution determined by the probability density function of density $f_\rho(\rho)$. Then, the random variable X has the distribution of density (Gerstenkorn and Śródka, 1972):

$$f(x) = f_\rho[\rho(x)]\left|\frac{d\rho(x)}{dx}\right| \tag{23a}$$

$$f(x) = \rho_o f_\rho(\rho = \rho_o x + \rho_o) \tag{23b}$$

After introducing a new random variable:

$$Y_1 = X^{1/2} \tag{24}$$

from which $x = y_1^2 = x(y_1)$.

The settling velocity is:

$$v_m = 5.33\sqrt{d_o}\; y_1 \tag{25}$$

The distribution of the random variable Y_1, analogically as above, is as follows:

$$f_1(y_1) = f[x(y_1)]\left|\frac{dx(y_1)}{dy_1}\right| \tag{26a}$$

$$f_1(y_1) = 2y_1 f(x = y_1^2) \tag{26b}$$

The following random variable Y_2 = 5.33 Y_1 $\left(y_1 = \frac{y_2}{5.33} = y_1(y_2)\right)$ has the probability density function:

$$f_2(y_2) = f_1[y_1(y_2)]\left|\frac{dy_1(y_2)}{dy_2}\right| \tag{27a}$$

$$f_2(y_2) = f_1\left(y_1 = \frac{y_2}{5.33}\right) \cdot \frac{1}{5.33} \tag{27b}$$

After the transformations the settling velocity as a random variable is expressed by the following formula:

$$V_{ms} = \sqrt{d_o}\, Y_2 \tag{28}$$

Consequently, the settling velocity of the set of monosized spherical particles has the frequency function:

$$h(v_{ms}) = f_2[y_2(v_{ms})]\left|\frac{dy_2(v_{ms})}{dv_{ms}}\right| \tag{29a}$$

$$h(v_{ms}) = f_2\left(y_2 = \frac{v_{ms}}{\sqrt{d_o}}\right)\frac{1}{\sqrt{d_o}} \tag{29b}$$

DISTRIBUTION OF SETTLING VELOCITY FOR MONOSIZED IRREGULAR PARTICLES

It was assumed that the sample of particles have the same value of the projection diameter $d_p = d_o$ while the particle shape coefficients constitute the random variables K_1 and K_2 of the frequency functions $w_1(k_1)$ and $w_2(k_2)$, respectively. Then, the frequency function of the random variable $Z = \frac{K_1}{K_2}$ is expressed by the following formula (Gerstenkorn and Śródka, 1972):

$$p_1(z) = \int k_2 w_2(k_2) w_1(zk_2) dk_2 \tag{30}$$

The following random variable $A = Z^{1/2}$ $(z = a^2 = z(a))$ has the frequency function determined by the formula:

$$p_2(a) = p_1[z(a)]\left|\frac{dz(a)}{da}\right| \tag{31a}$$

$$p_2(a) = 2ap_1(z = a^2) \tag{31b}$$

The random variable of settling velocity of monosized irregular particles, after the above transformations, is :

$$V_{mn} = V_{ms} \cdot A \tag{32}$$

As it can be seen from Eq. 32, the random variable of settling velocity of irregular particles is the product of the random variable of settling velocity of monosized spherical particles and the random variable being the function of particle shape coefficients. Consequently, the function of distribution density of settling velocity of monosized irregular particles is (Gerstenkorn and Śródka, 1972):

$$h(v_{mn}) = \int_{v_{s\,\min}}^{v_{s\,\max}} h(v_{ms}) p_2\left(a = \frac{v_{mn}}{v_{ms}}\right)\frac{1}{v_{ms}} dv_{ms} \tag{33}$$

EXPERIMENTAL

A narrow 8-10 mm size fraction was fed into the Allmineral jig. In this sample the densimetric analyses were performed. Moreover, by means of the image analysis, the distribution of projection diameter was determined as well as the distributions of volume and dynamic shape coefficients. In order to calculate the dynamic shape coefficient, the photographs of particles were taken with a digital camera in the most stable position. Next, using the image analysis software, the projection areas and perimeters of individual particles were calculated. Applying Eq. 14, the sphericity coefficients ϕ and projection diameters d_p were fixed, according to Eq. 9. The dynamic shape coefficient k_2 was calculated using dependence shown in Fig. 12. The volume shape coefficient k_1 was determined according to the volumetric method, consisting in the measurement of density of individual particles with a pycnometer and in calculating their volumes. The shape coefficient k_1 was calculated using Eq. 18.

MEASUREMENT RESULTS AND DISCUSSION

DISTRIBUTION OF PARTICLE DENSITIES

The cumulative distribution function of particle density, according to the dispersive particle model (Brożek, 1995), is expressed by a two-parameter function of gamma distributions family. As it will be shown further, that it is the Weibull distribution which cumulative distribution function and frequency function are as follows:

$$F_\rho(\rho) = 100\left\{1 - \exp\left[-\left(\frac{\rho}{\rho_c}\right)^{k_n}\right]\right\} \tag{34}$$

$$f_\rho(\rho) = \frac{100k}{\rho_c^{k_n}} \rho^{k_n - 1} \exp\left[-\left(\frac{\rho}{\rho_c}\right)^{k_n}\right] \tag{35}$$

where: ρ_c – characteristic density $(F_\rho(\rho = \rho_c) = 63.21\,\%)$, k_n – non-homogeneity coefficient.

On the basis of empirical data on the densimetric analysis, the coordinates of the cumulative distribution function of density were calculated and, next, Weibull's distribution was fitted to the empirical date. Distribution parameters ρ_c and k_n were obtained from this fitting. Figure 1 presents the model dependence with marked empirical values. The compatibility of the model with the experiment was estimated, calculating the index of curvilinear correlation. Its high values (R > 0.99) prove a good compatibility of the model distribution function with the empirical date.

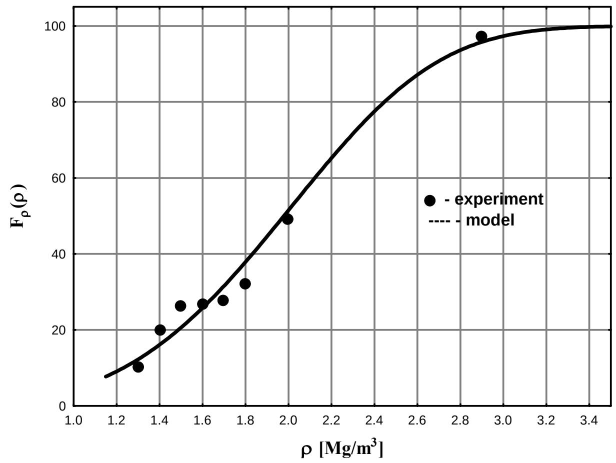

Fig. 1. Distribution of particle densities for size fraction 8 – 10 [mm], $\rho_c = 2170$ [kg/m^3]; $k_n = 3.97$

DISTRIBUTION OF PROJECTION DIAMETER

Figure 2 presents the histogram of distribution of projection diameter.

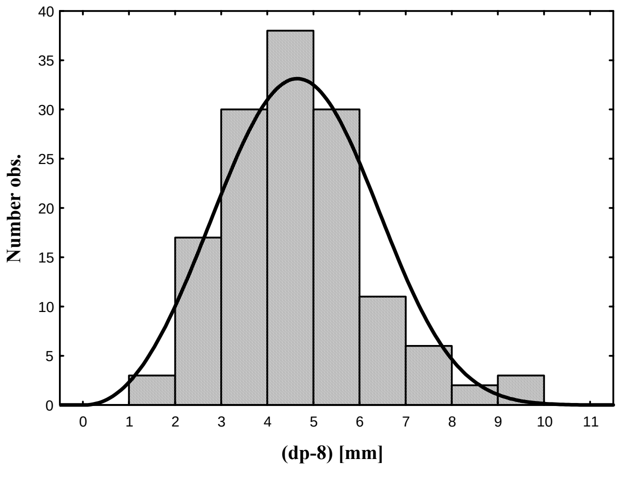

Fig. 2. Histogram of distribution of projection diameter of 8 -11 [mm] size fraction: $d_{cp} = 13.2$ [mm]; $k_p = 3.17$; $b_p = 8$ [mm]; $\bar{d}_p = 12.8$ [mm]

By means of the Statistica program the frequency function of projection diameter was fitted to the histogram. The best fitting was obtained for Weibull's distribution in the form:

$$g_p(d_p) = \frac{100k_p}{d_{cp}^{k_p}} d_p^{k_p-1} \exp\left[-\left(\frac{d_p - b_p}{d_{cp} - b_p} \right)^{k_p} \right] \tag{36}$$

where: d_c – characteristic value of projection diameter, k_p – non-homogeneity coefficient, b_p – value of shift of random variable.

For the description of Fig. 2 the parameters of Weibull's distribution together with the value of shift of random variable b_p are given. The average value of projection diameter is calculated from the following formula (Gerstenkorn and Śródka, 1972):

$$\overline{d}_p = d_{op} \Gamma\left(\frac{1}{k_p} + 1 \right) + b_p \tag{37}$$

The average value $\overline{d}_p = 12.8$ mm was assumed as the size of spherical particle in calculating the distribution of settling velocity of spherical particles.

DISTRIBUTION OF SHAPE COEFFICIENTS

Figures 3 and 4 show the histograms of distributions of random variables K_1 and K_2.

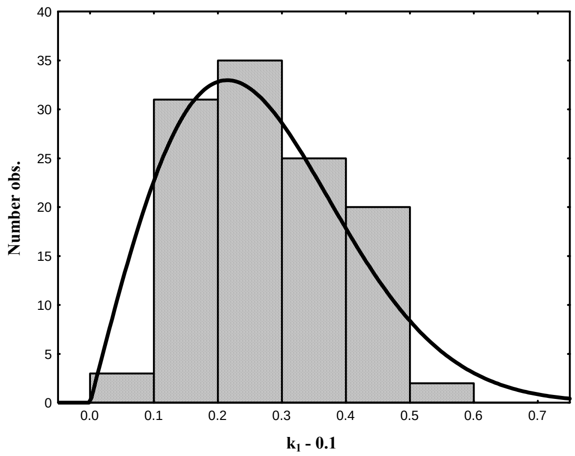

Fig. 3. Histogram of distribution of volume shape coefficient k_1, $\lambda_1 = 11$

The investigations of other authors indicate that the distributions of shape coefficients are of the gamma type (Hodenberg, 1998; Stark and Muller, 2005). Because of this, Rayleigh's and Weibull's distributions were fitted to these histograms. The statistical evaluation of compatibility of model distributions with empirical date was the same for both distribution types and, consequently, Rayleigh's distributions of shape coefficients were accepted and used in further considerations. Their general equations are as follows:

$$w_1(k_1) = 2\lambda_1 k_1 \exp(-\lambda_1 k_1^2) \quad (38)$$

$$w_2(k_2) = 2\lambda_2 k_2 \exp(-\lambda_2 k_2^2) \quad (39)$$

where λ_1 and λ_2 are distribution parameters.

The values λ_1 = 11 and λ_2 = 0.015 for distribution parameters were obtained from the fitting to empirical distributions with the Statistica program.

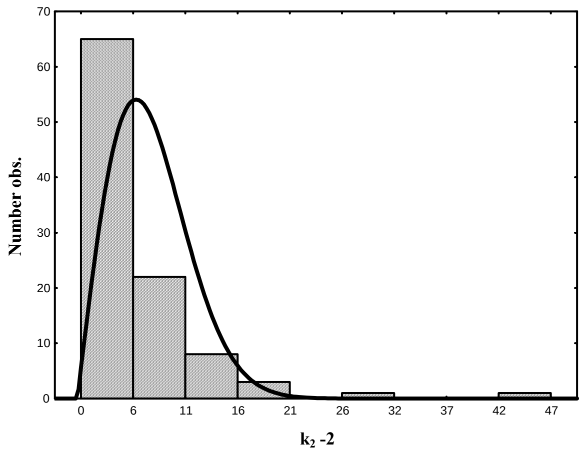

Fig. 4. Histogram of distribution of dynamic shape coefficient k_2 , λ_2 = 0.015

DISTRIBUTION OF SETTLING VELOCITY OF SPHERICAL PARTICLES

Applying the Eq. 29 and the algorithm of chapter 3, the frequency function and the cumulative distribution function of settling velocity of spherical particles were calculated.

$$h(v_{ms}) = \frac{200k_n}{(v_{msc}^2 + 28.41d_o)^{k_n}} v_{ms}(v_{ms}^2 + 28.41d_o)^{k_n - 1} \times \times \exp\left[-\left(\frac{v_{ms}^2 + 28.41d_o}{v_{msc}^2 + 28.41d_o}\right)^{k_n}\right] \quad (40)$$

$$H(v_{ms}) = 100\left\{1 - \exp\left[-\left(\frac{v_{ms}^2 + 28.41d_o}{v_{msc}^2 + 28.41d_o}\right)^{k_n}\right]\right\} \quad (41)$$

where: $d_o = \overline{d}_p = 12.8$ mm – diameter of sample particles, while v_{msc} is:

$$v_{msc} = \sqrt{d_o}\, y_{2c} = 5.33\sqrt{d_o \frac{\rho_c - \rho_o}{\rho_o}} \tag{42}$$

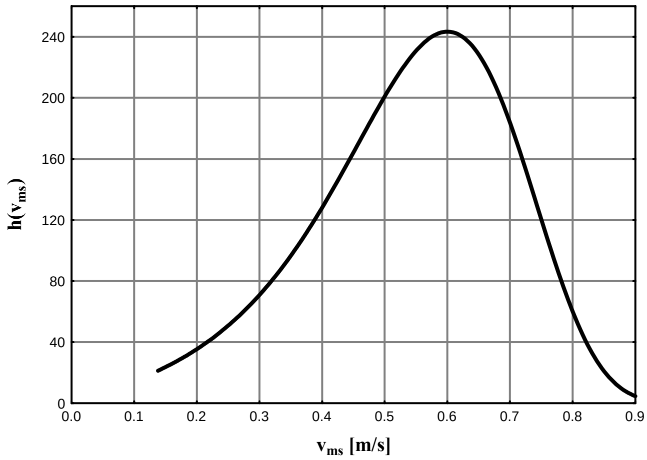

Fig. 5. Frequency function of particle settling velocity for size fraction 8 -10 [mm]

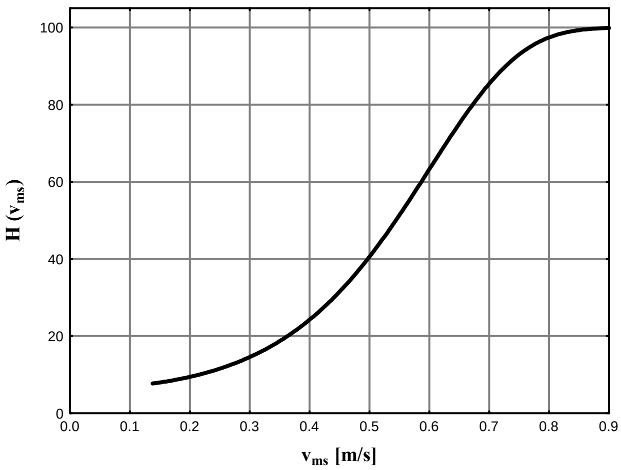

Fig. 6. Cumulative distribution function of settling velocity of particles for size faction 8 -10 mm, v_{msc} = 0.599 m/s, k_n = 3.97

As it can be seen from Eq. 41 – 42 that the distribution of settling velocity in this case is expressed by the parameters of distribution of particles density and their diameter. Analogically as for the density distribution, this is also Weibull's distribution. The parameters of settling velocity distribution are constituted by non-homogeneity coefficient k_n and characteristic velocity v_{msc}, calculated according to Eq. 42. The density ρ_o of liquid, occurring in this formula, which is transferred to the jig chamber, is higher than water density and equals $\rho_o=1093$ kg/m^3 because it is recirculated from Dorr's settling tank. Figures 5 and 6 present the frequency and cumulative distribution function of settling velocity, respectively, in the sample of spherical particles.

DISTRIBUTION OF SETTLING VELOCITY OF IRREGULAR PARTICLES

According to Eq. (33) and the algorithm of chapter 4 the frequency function of settling velocity of irregular particles is expressed by the following formula:

$$h(v_{mn}) = 1941 \int_0^{0.88} \left(v_{ms}^2 + 0.36\right)^{2.97} exp\left[-\left(\frac{v_{ms}^2 + 0.36}{0.72}\right)^{3.97}\right] \frac{v_{mn}^3 \cdot v_{ms}^5}{(0.015 v_{ms}^4 + 11 v_{mn}^4)^2} dv_{ms} \tag{43}$$

As it results from Eq. 43 that the distribution of settling velocity is openly dependent on the distribution of particle shape coefficients. Because of complexity of that integral, integration was performed by the numerical method. Figures 7 and 8 present, respectively, the frequency and cumulative distribution function of settling velocity of irregular particles. The comparison of both graphs indicate that, after considering the distribution of shape coefficients, the shape of the frequency function changes (distribution asymmetry changes from negative to positive), the values of the most probable velocity decreases from 0.6 m/s to 0.084 m/s and also the maximum value of settling velocity decreases from 0.88 m/s to 0.4 m/s. Consequently, the flat particles of higher density than the analogical particles of lower density and lower asymmetry will be grouped in the light product. This conclusion is in agreement with the observed experimental facts (Ferrara et al., 2000; Ociepa and Mączka, 2000).

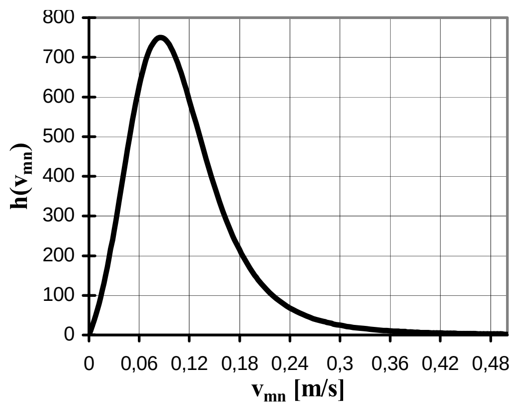

Fig. 7. Frequency function of settling velocity of irregular particles

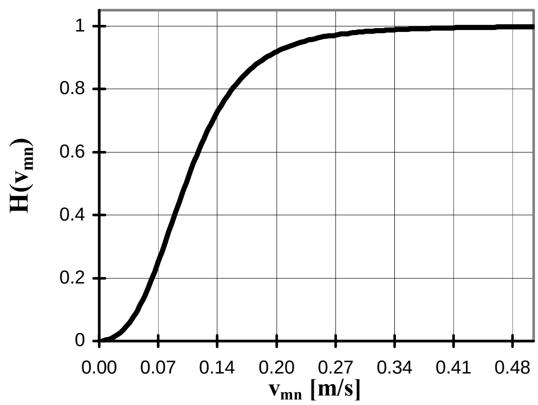

Fig.8. Cumulative distribution function of settling velocity of irregular particles

EFFECT OF PARTICLE SHAPE UPON SEPARATION EFFICIENCY

The indicators based on the partition curves, i.e. probable error and imperfection, are the most popular and most often applied indicators of evaluation of separation efficiency. Belugu (1959, according to Samylin et al., 1976) gave the following empirical dependence to evaluate the effect of distribution of particle density in the feed:

$$I = I_o + 0.021 tg\alpha \tag{44}$$

where: I_o – value of imperfection, characteristic for a given jig, $tg\alpha = k_I \dfrac{\Delta\gamma(\Delta\rho)}{\Delta\rho}$, k_I – coefficient of scale, $\Delta\rho = \pm 100$ kg/m^3 – range of density around partition density, $\Delta\gamma(\Delta\rho)$ – content of particles in the feed of the density range $(\rho_r - \Delta\rho,\ \rho_r + \Delta\rho)$ in %. To estimate the effect of shape coefficients distribution upon separation efficiency the authors used the dependence analogical to dependence in Eq. 44 in relation to settling velocity as the argument of separation:

$$\Delta I_{mn} = I_{mn} - I_o = a\, tg\alpha_{mn} \tag{45}$$

where: ΔI_{mn} – change of imperfection, I_o and a – constants, $tg\alpha_{mn} = \dfrac{\Delta\gamma_{mn}(\Delta v)}{\Delta v}$, $\Delta\gamma_{mn}(\Delta v)$ – content of particles in the feed of the settling velocity range $(v_r - \Delta v,\ v_r + \Delta v)$, $\Delta v = \pm 0.04$ m/s – range of settling velocity around the partition velocity.

The width of Δv range was selected by analogy to the width of density range $\Delta\rho = \pm 100$ kg/m^3 $\cong \frac{1}{8}(2000-1250)$ kg/m^3 which Belugu proposed in Eq. 44. According to Fig. 8 $\Delta v = \frac{1}{8} \cdot 0.32$ mm.

Analogically to Eq. 45, for the separation of spherical particles the equation can be written as:

$$\Delta I_{ms} = I_{ms} - I_o = a\, tg\alpha_{ms} \quad (46)$$

And the values of this equation have a similar interpretation as in Eq. 45 only in relation to spherical particles. Dividing by sides Eq. 45 by Eq. 46 we obtain:

$$\frac{\Delta I_{mn}}{\Delta I_{ms}} = \frac{tg\alpha_{mn}}{tg\alpha_{ms}} = \frac{\Delta\gamma_{mn}(\Delta v)}{\Delta\gamma_{ms}(\Delta v)} \quad 47)$$

From Eq. 47, having the distributions of settling velocities of irregular and spherical particles in the feed, it is possible to calculate the relation of process imperfection changes during the separation of irregular and spherical particles in the same conditions. For the separation velocity determined from the equation of partition curve for waste of 8-10 mm size fraction the following value was obtained (Surowiak, 2007): v_r = 0.159 m/s. Respectively, from Fig. 8. $H(v_{mn} = 0.159) \cong 82\%$ and $\Delta\gamma_{mn}(\Delta v) \cong 28\%$. Next, for spherical particles, according to Fig. 6. for $H(v_{ms}) = 82\%$, velocity v_{ms} = 0.68 m/s. The content of spherical particles of velocity range (0.68 – Δv; 0.68 + Δv) in the feed is $\Delta\gamma_{ms}(\Delta v) \cong 16$ % . Respectively, from Eq. 47:

$$\frac{\Delta I_{mn}}{\Delta I_{ms}} = \frac{28}{16} = 1.75 \quad (48)$$

According to the obtained results, it can be stated that in the case of the tested coal sample the irregular shape of particles results in about 70% larger change of process imperfection on average than in the case of separation of a sample of monosized spherical particles under the same conditions, which is connected with the decrease of separation efficiency of irregular particles. This increase results from the decrease of difference of settling velocity of irregular particles in relation to the difference of settling velocity of spherical particles.

CONCLUSIONS

- The frequency function of settling velocity in the sample of monosized spherical particles is Weibull's distribution of negative asymmetry. On the other hand, however, in the case of monosized irregular particles (particles of the same projection diameter and distribution of shape coefficients), the asymmetry is positive. The irregular particles shape decreases the value of settling velocity. This increases the probable error as a result of the decrease of the non-homogeneity rate of velocity distribution in the sample.
- The irregularity of particle influences the separation efficiency measured by the imperfection change. The difference in particle settling velocity decreases with the growth of the particle irregularity rate (increase of dynamic shape coefficient and decrease of volume coefficient). This narrows the range of variation of settling velocity of irregular particles in relation to the analogical range for spherical particles, causing the increase of particles content in the range of settling velocity around partition velocity and, automatically, also the increase of imperfection.

ACKNOWLEDGEMENTS

This work was supported by a grant of the National Research Committee No. 4 T12A 006 29.

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Na prędkość opadania ziaren, będącą argumentem rozdziału w osadzarce, wpływają właściwości fizyczne (gęstość) oraz geometryczne (wielkość i kształt) ziaren. W artykule opracowano algorytm wyliczania rozkładu prędkości opadania ziaren w monodyspersyjnej próbce ziaren sferycznych i nieregularnych przy założeniu, że gęstość ziaren, ich wielkość i kształt stanowią zmienne losowe o określonych rozkładach . W oparciu o eksperyment przemysłowy wyliczono rozkłady prędkości opadania ziaren nieregularnych w wąskiej klasie ziarnowej. Wykonano pomiary i wykreślono histogramy rozkładów średnicy projekcyjnej, objętościowego i dynamicznego współczynnika kształtu. Dokładność rozdziału mierzono zmianą imperfekcji procesowej ziaren nieregularnych w stosunku do ziaren sferycznych wynikającą z rozkładu prędkości opadania ziaren.